

Greatly Enhanced Eccentricity Oscillations in Quadruple Systems Composed of Two Binaries: Implications for Stars, Planets, and Transients

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ABSTRACT

We study the orbital evolution of hierarchical quadruple systems composed of two binaries on a long mutual orbit, where each binary acts as a Kozai-Lidov (KL) perturber on the other. We find that the coupling between the two binaries qualitatively changes the behavior of their KL cycles. The binaries can experience coherent eccentricity oscillations as well as excursions to very high eccentricity that occur over a much larger fraction of the parameter space than in triple systems. For a ratio of outer to inner semi-major axes of 10 to 20, about 30 to 50% of equal-mass quadruples reach eccentricity $1 - e < 10^{-3}$ in one of the binaries. This is about 4 to 12 times more than for triples with equivalent parameters. Orbital “flips” and collisions without previous tidal interaction are similarly enhanced in quadruples relative to triples. We argue that the frequency of evolutionary paths influenced by KL cycles is comparable in the triple and quadruple populations even though field quadruples are a factor of ~ 5 less frequent than triples. Additionally, quadruples might be a non-negligible source of triples and provide fundamentally new evolutionary outcomes involving close binaries, mergers, collisions, and associated transients, which occur without any fine tuning of parameters. Finally, we study the perturbations to a planetary orbit due to a distant binary and we find that the fraction of orbital flips is a factor of 3 to 4 higher than for the corresponding triple system given our fiducial parameters with implications for hot Jupiters and star-planet collisions.

Key words: Binaries: close — binaries: general — planets and satellites: dynamical evolution and stability — stars: kinematics and dynamics

1 INTRODUCTION

The long-term orbital evolution of hierarchical triple systems of stars and planets is subject to Kozai-Lidov (KL) cycles (Kozai 1962; Lidov 1962) that transfer the angular momentum between the inner and outer orbits. As a result, high eccentricity and close pericenter passages of the inner binary can occur and physical effects that would not be possible otherwise can significantly change the binary dynamics. For example, tidal friction removes orbital energy and decreases the semi-major axis a of the inner binary, potentially explaining the existence of short-period stellar binaries (e.g. Mazeh & Shaham 1979; Tokovinin et al. 2006; Wu et al. 2007; Fabrycky & Tremaine 2007), hot Jupiters, and irregular satellites of planets (Nesvorný et al. 2003). The high eccentricities induced by KL cycles can also reduce the gravitational wave merger timescale of compact objects (Blaes et al. 2002; Miller & Hamilton 2002; Thompson 2011; Antonini & Perets 2012; Naoz et al. 2012a;

Shappee & Thompson 2013; Hamers et al. 2013; Antognini et al., in preparation).

The phenomenology of KL cycles is usually investigated at several levels of approximation ranging from analytic equations for the secular evolution of the orbital elements to fully numerical studies. In the simplest case of the secular evolution of a test particle subject to a three-body Hamiltonian expanded to quadrupole order (hereafter TPQ limit), the KL cycles occur only if the initial mutual inclination i between the inner and outer orbits is smaller than $\cos^2 i_K \equiv 3/5$ (e.g. Kozai 1962). The test particle achieves a maximum eccentricity of $e_{\max} = \sqrt{1 - (5/3) \cos^2 i}$. In more realistic settings, the dynamics become more chaotic and the restrictions of the simplest case, especially the maximum eccentricity, do not apply. For example, higher-order terms of the Hamiltonian give rise to the eccentric Kozai mechanism (Ford et al. 2000; Naoz et al. 2011, 2013; Lithwick & Naoz 2011; Katz et al. 2011), which operates for unequal-mass inner binaries with eccentric outer orbits, giving rise to arbitrarily high eccentricities and flips of the orientation of the inner orbit. In addition, Katz & Dong (2012) showed with direct integration that for a small fraction of triples the angular momen-

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tum of the inner binary can go from a finite value to essentially zero in one orbit, which can produce stellar collisions with no prior tidal or gravitational wave interaction. This mechanism can perhaps produce supernovae Ia by colliding two white dwarfs. Although the full parameter space exploration of KL cycles in triple systems with direct three-body integration remains to be explored, the basic principle that systems with lower initial $|\cos i|$ reach higher eccentricities is still valid.

One of the possible stable hierarchies of quadruple systems are two binaries on a mutual orbit. In these systems, each binary acts as a distant perturber inducing KL cycles on the other binary. Since the perturber is not a point mass, the evolution of such quadruple systems can in principle differ from a combination of two uncoupled three-body KL processes. KL cycles in quadruples have not been studied previously. Recent discoveries of double close eclipsing binaries with periods very close to a 3:2 ratio (Čagaš & Pejcha 2012; Kołaczowski et al. 2013) indicate that quadruples might exhibit new and unexplained dynamics that might be related to KL cycles. Moreover, a quadruple with mutual KL cycles has also been proposed as the origin of Tycho B as a surviving companion to the Tycho SN progenitor (Thompson & Gould 2012; Kerzendorf et al. 2012).

In this paper, we show that for a significant part of the parameter space the evolution of two binaries on a mutual orbit does not reduce to two independent systems consisting of a binary and distant point-like perturber. As a result of more degrees of freedom and the mutual coupling, the two binaries evolve in concert and experience excursions to very high eccentricities that would not be achieved otherwise. In Section 2, we describe our calculations. In Section 3, we compare the evolution of quadruples to that of triple stars. In Section 4, we study collisions in quadruple systems. In Section 5, we summarize our results and discuss the implications of our findings for the evolution of stars and planets.

2 CALCULATIONS

We modified the N -body code *fewbody* (Fregeau et al. 2004) to simulate the evolution of quadruple stars. The integrator of *fewbody* is based on the GNU Scientific Library (Galassi et al. 2011). We tested Runge-Kutta Prince-Dormand (rk8pd) and the Bulirsch-Stoer method of Bader and Deuflhard (bsimp) and several conditions for adaptive timestep algorithm, all of which give qualitatively similar results for a few test cases. We do all of the calculations shown here with the default adaptive timestep algorithm with rk8pd, which is much faster and conserves energy better than bsimp. We also perform the calculations using Kustaanheimo-Stiefel regularization (Heggie 1974; Mikkola 1985), which improves the treatment of close encounters and provides well-behaved long-term energy conservation.

The dynamics of the four-body problem is inherently chaotic and eventually the numerical orbits deviate from the true ones with the same initial conditions. For calculations with absolute and relative accuracies between 10^{-11} and 10^{-14} we find that the system trajectories typically start to deviate from each other after a few times 10^4 orbits of either of the inner binaries. We continue the integration for longer times since we are interested in the statistical behavior of quadruple systems. We perform all our calculations with absolute and relative integration accuracy of 10^{-12} and we find that this was likely an unnecessarily high accuracy since the occurrence of $1 - e < 10^{-3}$ (see Section 3) is higher only by 1 to 3% for accuracy of 10^{-11} while it is not converged for accuracy of

10^{-10} . We also verified our calculations by performing simulations of triple stars, where the properties of KL cycles are known and we successfully reproduce them. Our calculations conserve total angular momentum to about 10^{-9} and total energy to about 10^{-8} of the initial value for our typical relative separations ($a_{AB}/a_A \approx 10$). However, for large relative separations ($a_{AB}/a_A \gtrsim 50$) or very different masses of the components, energy conservation is worse. We monitor the energy conservation of the calculations and remove all instances when the final relative energy conservation is worse than 10^{-5} . However, for our fiducial choice of accuracy, the constraint on energy conservation is violated by only several runs in many thousands and thus the statistics of our results are not affected.

Since the parameter space of quadruple systems composed of binaries A and B on a mutual orbit AB is vast, we set up the calculations by specifying the masses of all four bodies ($m_A = m_1 + m_2$, $m_B = m_3 + m_4$), and semi-major axes (a_A , a_B , a_{AB}) and eccentricities (e_A , e_B , e_{AB}) of the three orbits. We systematically vary the angles i_A and i_B between the angular momenta of orbits A and B with respect to their mutual orbit AB . The remaining parameters (arguments of pericenter, longitudes of ascending node, and mean anomalies) are chosen randomly. Typically, we perform 100 different random initializations for each pair of $(\cos i_A, \cos i_B)$.

Due to computational limitations, we run the calculations for 300 Kozai times t_K (Holman et al. 1997; Innanen et al. 1997) of system A defined as

$$\begin{aligned} t_K &= \frac{4}{3} \left(\frac{a_A^3 m_A}{G m_B^2} \right)^{1/2} \left(\frac{a_{AB}(1 - e_{AB}^2)^{1/2}}{a_A} \right)^3, \\ &\simeq 1.1 \times 10^5 \text{ yr} \left(\frac{a_A}{20 \text{ AU}} \right)^3 \left(\frac{m_A}{m_B} \right)^{1/2} \left(\frac{m_B}{2 M_\odot} \right)^{-1/2} \times \\ &\quad \times \left(\frac{a_{AB}/a_A}{20} \right)^3 (1 - e_{AB}^2)^{3/2} \end{aligned} \quad (1)$$

Higher-order effects in KL cycles become important only on longer timescales than t_K . For example, the secular evolution of the eccentric Kozai mechanism occurs on a timescale of $t_K/\epsilon_{\text{oct}}$ (Lithwick & Naoz 2011; Naoz et al. 2011, 2013; Katz et al. 2011), where ϵ_{oct} is the relative strength of the octupole to quadrupole term in three-body Hamiltonian expansion:

$$\epsilon_{\text{oct}} = \left(\frac{m_1 - m_2}{m_1 + m_2} \right) \frac{a_A}{a_{AB}} \frac{e_{AB}}{1 - e_{AB}^2}. \quad (2)$$

The eccentric Kozai mechanism disappears for equal-mass binaries or for circular outer orbits in the secular approximation. Although Equations (1) and (2) are informative for triple systems, it is unclear how they generalize to the quadruple case. Nonetheless, we use t_K to determine the length of our simulation runs and ϵ_{oct} serves as a characteristic of the triple dynamics that we use as a basis for understanding some quadruple phenomenology we observe. To get our results in finite time with the computational resources available to us, we limit the duration of any single calculation to 1500 s. This limitation becomes important only for $a_{AB}/a_A \gtrsim 20$. The quadruple dynamics is different from two uncoupled three-body KL cycles on timescales shorter than the limits we impose on the duration of the calculation. Thus, our results should be very close to the “true” results obtained with much longer calculation times, although technically we obtain lower bounds on the true result. We run the reference triple calculations for the same number of t_K as the quadruple calculations. In a number of calculations presented in this paper, we stop the integration when a certain value of eccentricity is reached or when the orbital orientation flips.

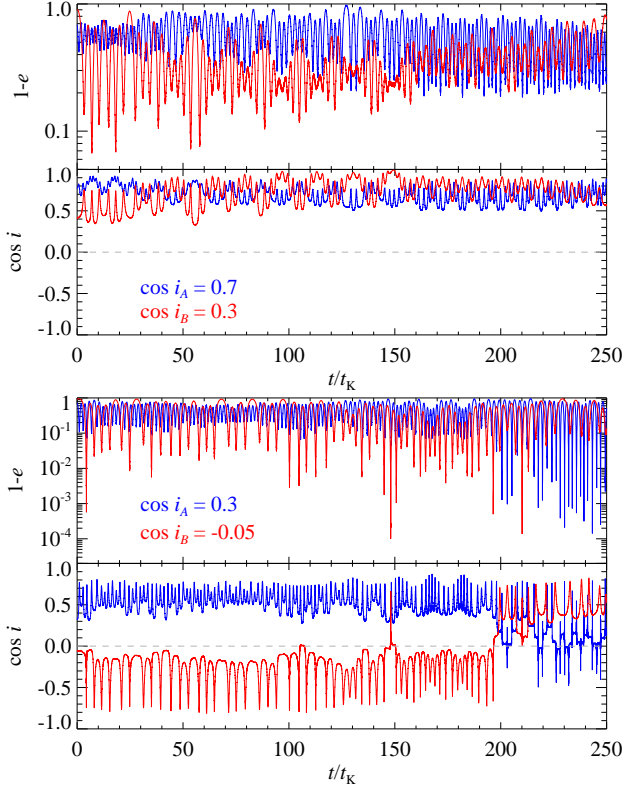


Figure 1. Illustration of two possible behaviors of eccentricities e and binary orientations $\cos i$ (measured with respect to the mutual orbit) in quadruple systems. The two upper panels show the two binaries evolving in concert and the two lower panels depict evolution where both binaries flip orbital orientation and go through a high-eccentricity phase. The behavior of both systems differs qualitatively from the expectation of two uncoupled triple systems. The blue lines are for system A with $m_1 = m_2 = 1 M_\odot$, $a_A \approx 0.57$ AU, and $e_A = 0.3$, while the red lines are for system B with $m_3 = m_4 = 1 M_\odot$, $a_B \approx 0.35$ AU, and $e_B = 0.1$. The initial inclinations are given in the plots. The mutual orbit has $a_{AB} = 5$ AU and $e_{AB} = 0.3$.

3 COMPARISON WITH TRIPLES

In this Section, we investigate the properties of the quadruple dynamics and we show that it is not simply a superposition of two independent KL cycles. Our fiducial calculation is for equal-mass binaries ($m_1 = m_2 = m_3 = m_4 = 1 M_\odot$) to eliminate the influence of eccentric Kozai mechanism for the equivalent triple systems ($\epsilon_{\text{Oct}} = 0$), though we consider unequal-mass binaries in Sections 3.2 and 3.3. Summary of our quadruple calculations is given in Table 1.

Figure 1 illustrates two examples of behavior we observe in our calculations.¹ The two upper panels show an initial phase of chaotic and apparently independent evolution of the two binaries followed by their long-term synchronized evolution with eccentricity modulations occurring in opposite phases. Although the synchronization eventually ceases, we speculate that such mutually coherent evolution is a necessary element of any theory attempting to explain the near-integer period ratios in CzeV343 (Čagaš & Pejcha

¹ We note that the evolution of the three orbits is very close to secular in the sense that the standard deviations of the three semi-major axes, and angular momentum and eccentricity of the outer orbit around the initial values are $\lesssim 0.5\%$.

2012) and BI 108 (Kołaczowski et al. 2013). However, the current parameters of these observed systems dictate that such a theory must include tidal forces; we plan to investigate the interplay between KL cycles and tidal friction in a future work.

The two lower panels of Figure 1 show a qualitatively different behavior: binary A initially shows only small eccentricity variations compatible with its relatively large initial inclination $\cos i_A = 0.3$, while binary B reaches eccentricities $1 - e_B \approx 10^{-2}$ to 10^{-3} , which is expected from its initial inclination $\cos i_B = -0.05$. At $t \approx 148 t_K$, binary B briefly flips the direction of its orbit and reaches $1 - e_B \approx 10^{-4}$. The evolution continues in a similar fashion until $t \approx 200 t_K$ when both binaries flip their orbit orientation. As a result, binary A goes through many epochs of high eccentricity ($1 - e_A \approx 10^{-4}$) and the eccentricity oscillations in binary B become much shallower, reaching only $1 - e_B \approx 0.1$. We emphasize that in this calculation system A had a moderate initial value of inclination, $\cos i_A = 0.3$, and that a triple system analogous to system A with binary B replaced by a point mass would not achieve such high eccentricities. In the TPQ limit ($m_1 \equiv m_2$ and hence $\epsilon_{\text{Oct}} = 0$), the maximum eccentricity of binary A would be $e_{A,\text{max}} \approx \sqrt{1 - (5/3)0.3^2} \approx 0.92$ (Kozai 1962; Innanen et al. 1997; Naoz et al. 2013).

To characterize the quadruple properties more completely, we performed calculations spanning the full range of $(\cos i_A, \cos i_B)$. For each calculation, we recorded the highest achieved eccentricity of each binary and we show the median of these values as a function of $(\cos i_A, \cos i_B)$ using contours in Figure 2. The region of high eccentricities, $1 - e \lesssim 10^{-3}$, extends well beyond $\cos i_A \approx 0$ and $\cos i_B \approx 0$ and thus a significant fraction of both binaries in the quadruple experience high eccentricities. In the TPQ limit, we would expect $\sqrt{(3/5)(1 - 0.999^2)} \approx 3\%$ of all binaries to reach such an eccentricity. By direct integration of triple orbits we obtain about 7% for $a_{AB}/a_A \approx 10$. This is still much smaller than for quadruples, where we found that 36 and 53% of quadruples reach high eccentricity in binaries A and B , respectively (see Section 3.1). The region of high eccentricities is approximately bounded by the Kozai angle $\cos i_K = \pm\sqrt{3/5}$ of each binary. The asymmetry of the contours suggests that the strongest effect is achieved when the binaries were initially on mutually retrograde orbits. The fraction of high-eccentricity systems is higher for binary B than for binary A . The reason is that the perturbation to binary B from binary A and vice versa is proportional to a power of a_A/a_{AB} and a_B/a_{AB} , respectively, and thus with the same masses and a_{AB} , the high-order perturbations induced by A on B are bigger than those of B on A .

Figures 1 and 2 indicate that the parameter space of high eccentricities is potentially much larger in quadruples than in triples due to the mutual coupling of the two binaries. Now we numerically explore the dependence of the coupling on semi-major axes ratios, mass ratios, and eccentricities, and compare it to triples.

3.1 Dependence on semi-major axes ratio

We show in Figure 3 the fraction of quadruples with $1 - e < 10^{-3}$ as a function of the ratio of semimajor axes a_{AB}/a_A and a_{AB}/a_B . The orbital evolution time of our simulations is $300 t_K \approx 14$ Myr for $a_A = 20$ AU and a perturbing binary on $a_{AB}/a_A \sim 15$ orbit. We assume that the inclinations of both binaries are distributed uniformly in $\cos i$. To directly compare the quadruples with the triples, we performed a series of calculations where we replaced either of the binaries in the quadruple with a point mass with the same total mass. These triple have an equal-mass inner binary and thus the ec-

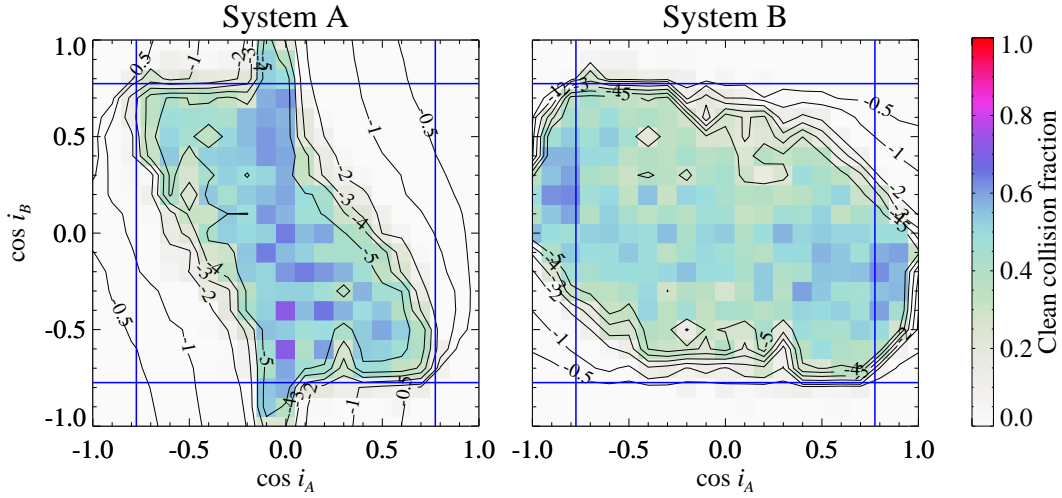


Figure 2. KL cycles in quadruple systems composed of two binaries. At each pair of initial inclinations $(\cos i_A, \cos i_B)$, we performed 100 randomly initialized calculations and recorded the highest eccentricity and the occurrence of clean collisions. The solid lines show contours of median value of the highest achieved eccentricity and are labelled by $\log_{10}(1 - e)$. The color corresponds to the fraction of calculations that experienced a “clean” collision (pericenter distance at least four times smaller than any of the previous pericenters) as set by the colorbar. The blue horizontal and vertical lines mark the Kozai angle $\cos i_K = \pm\sqrt{3}/5$ for each binary. The system parameters are given in Table 1 as F6.

Table 1. Summary of quadruple system calculations.

Name	a_A [AU]	e_A	a_B [AU]	e_B	a_{AB} [AU]	e_{AB}	$(m_1, m_2, m_3, m_4) [M_\odot]$	N	f_A	f_B
F6	0.573	0.3	0.351	0.1	6	0.3	(1, 1, 1, 1)	441	0.358	0.530
F12	0.573	0.3	0.351	0.1	12	0.3	(1, 1, 1, 1)	251	0.146	0.208
A6	0.573	0.2	0.573	0.2	6	0.3	(1, 1, 1, 1)	121	0.579	0.581
A6e0	0.573	0.2	0.573	0.2	6	0	(1, 1, 1, 1)	121	0.579	0.581
A12e0	0.573	0.2	0.573	0.2	12	0	(1, 1, 1, 1)	121	0.463	0.467
A24e0	0.573	0.2	0.573	0.2	24	0	(1, 1, 1, 1)	121	0.244	0.247
B6	0.691	0.3	0.319	0.1	6	0.3	(2, 1.5, 1, 0.5)	121	0.203	0.776
SJSS	0.428	0.1	0.573	0.3	6	0.3	$(1, 9.5 \times 10^{-3}, 1, 1)$	441	—	—

N is the number of points in the $(\cos i_A, \cos i_B)$ plane, which might not be distributed uniformly. Each point in $(\cos i_A, \cos i_B)$ plane had 100 calculations with randomly initialized arguments of pericenter, longitudes of ascending nodes, and mean anomalies. f_A (f_B) is the fraction of quadruples with binary A (binary B) reaching $1 - e < 10^{-3}$. Symbols “—” indicate simulations where this quantity was not tracked.

centric Kozai mechanism does not operate. We see from Figure 3 that for $a_{AB}/a_A \approx 10$ ($a_{AB}/a_B \approx 17$) 36% (53%) of quadruples reach $1 - e < 10^{-3}$ for binary A (B). This is by a factor of 4.8 (10.0) higher fraction than for otherwise equivalent triples. For wider separation of the binaries, the enhancement decreases: for $a_{AB}/a_A \approx 21$ ($a_{AB}/a_B \approx 34$) we find high-eccentricity quadruple fractions higher by a factor of 2.8 (4.9) than for equivalent triples. Wider relative separations are difficult to investigate numerically, but it is likely that the fraction of quadruples experiencing high eccentricity asymptotically approaches the fraction of triples as the ratio of semimajor axes increases. Based on Figure 3, we estimate the high-eccentricity fraction of quadruples is significantly enhanced with respect to triples for $a_{AB}/a_A \lesssim 50$.

3.2 Dependence on mass ratios

So far we have discussed only quadruples composed of equal-mass stars. The strength of the interaction depends also on $q_A = m_2/m_1$, $q_B = m_4/m_3$, and $q_{AB} = m_A/m_B$. We expect that the mutual interaction vanishes for two stars orbited by a test particle

($q_A \rightarrow 0, q_B \rightarrow 0$), when the dynamics reduces to two independent KL cycles. To see how non-equal mass quadruples behave we ran a calculation with $q_A = 3/4$, $q_B = 1/2$, $q_{AB} = 7/3$ (entry B6 in Table 1). As expected, the fraction of calculations where system A reached high eccentricity is lower (20%) than in the fiducial case because of the lower mass of the perturbing binary m_B . Interestingly, the fraction increases for binary B (77%) and essentially all systems with $\cos^2 i_B \leq \cos^2 i_K = 3/5$ reach very high eccentricity. The reason is again that a_A/a_{AB} is significantly larger than a_B/a_{AB} , and also $m_A > m_B$. In triple systems with different inner binary masses, eccentric Kozai mechanism becomes important ($\epsilon_{\text{oct}} > 0$) and induces high eccentricities over a wider range of $\cos i$ than in equal-mass systems (Ford et al. 2000; Naoz et al. 2011, 2013; Lithwick & Naoz 2011; Katz et al. 2011). We verified by direct integration of triple systems that for our unequal-mass system, the fraction of quadruples reaching $1 - e < 10^{-3}$ is a factor of about 3.0 (binary A) to 8.2 (binary B) higher than for corresponding triple systems defined by replacing one of the binaries by a point mass.

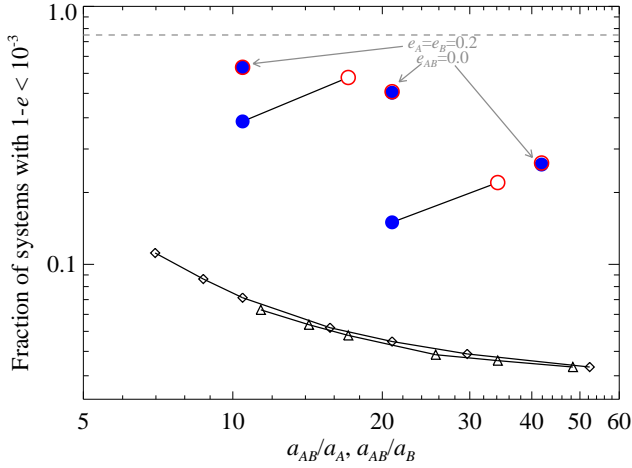


Figure 3. Fraction of systems with $1 - e < 10^{-3}$ as a function of the semimajor axis ratio showing that more quadruples than triples reach high eccentricity. The circles connected by a line indicate results for the two binaries A (solid blue) and B (open red) of quadruple systems. The triangles and diamonds connected by a line indicate results from systems where either binary A or B was replaced by a point mass with the same total mass. The fiducial system parameters are given in Table 1 in lines F6 and F12. Three overlapping circles labelled with grey arrows denote binaries with $a_A \equiv a_B$ (entries A6e0, A12e0, and A24e0 in Table 1) The grey horizontal dashed line indicates fraction of binaries with $\cos^2 i \leq \cos^2 i_K$ assuming uniform distribution of $\cos i$.

3.3 Planets and orbital flips

Systems of a star and planet on a mutual orbit around a distant binary are an important example of quadruples with an extreme mass ratio. The planetary orbit can experience the eccentric Kozai mechanism leading to high eccentricity and orbital flips, which can potentially explain the abundance of observed retrograde hot Jupiters (Naoz et al. 2011, 2013; Lithwick & Naoz 2011; Katz et al. 2011).

In Figure 4, we show the flip fraction of planet orbits evaluated as a change of sign of the projection of the star plus planet angular momentum to the total angular momentum of the quadruple. For the parameters of this particular system and assuming a uniform distribution for $\cos i$ of both binaries, we find that about 66% of all planetary orbits flip their orientation. How does this compare to triple systems where the distant binary is replaced by a single star? The flip fraction of triples presented in the upper panel of Figure 4 shows clear signs of the eccentric Kozai mechanism as the broad component of the peak at $-0.5 \lesssim \cos i_A \lesssim 0.3$. Nonetheless, the total flip fraction of $\sim 21\%$ is still ~ 3.2 times lower than for the quadruple system. The eccentric Kozai mechanism operates on a longer timescale, $t_K/\epsilon_{\text{oct}} \approx 40t_K$, for our choice of $a_A/a_{AB} \approx 0.072$ and $e_{AB} = 0.3$ and we thus ran our triple calculation for $1500t_K \approx 38t_K/\epsilon_{\text{oct}}$. For our fiducial calculation length of $300t_K \approx 8t_K/\epsilon_{\text{oct}}$, the flip fraction was about 2% lower, suggesting that our comparison of quadruples and triples is robust.²

² The triple flip fraction in Figure 4 does not look completely converged despite 100 random initializations at each $\cos i_A$. It is possible that a significantly higher fraction of triples with $-0.5 \lesssim \cos i_A \lesssim 0.3$ would eventually flip their orbits given integration times that are orders of magnitude longer than what we are able to achieve. The flip fraction would be $\sim 40\%$ if *all* such orbits flipped, but this is still 1.7 times less than what we obtain for quadruples. It is also quite likely that the quadruple flip fraction would increase as well for very long integration times.

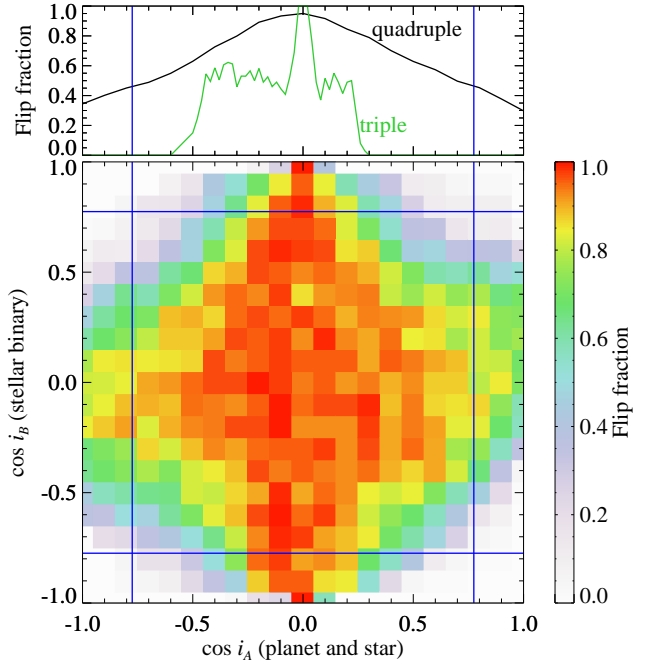


Figure 4. Orbital flip fractions for star plus planet systems orbiting another binary. The lower panel shows the flip fraction as a function of the inclination of the star plus planet ($\cos i_A$) and star plus star ($\cos i_B$) binaries. The upper panel shows the flip fraction integrated over all orientations of the star+star binary (black line) in comparison to a triple system, which has the outer binary replaced by a point mass with the same total mass (green line). The system parameters are given in Table 1 as sjss.

Although the system in Figure 4 is not representative of typical hot Jupiter systems (e.g. Wu et al. 2007; Naoz et al. 2012b) and a full exploration of the parameter space is beyond the scope of this paper, our results suggest that it is much easier to place a planet on a close and possibly retrograde orbit around a star that orbits a relatively distant stellar binary. Similarly, star-planet collisions are also more likely in quadruple systems than in triples.

3.4 Dependence on eccentricities

Finally, the coupling between the two binaries depends also on the remaining orbital parameters that we either specified explicitly as e_A , e_B , e_{AB} , or marginalized over by randomly initializing the calculations (arguments of pericenter, longitudes of ascending node, mean anomalies). In order to characterize the effect of the initial eccentricities of the orbits, we performed several calculations with various choices of eccentricities and we did not find any significant changes (for example A6 and A6e0 in Table 1), except that high initial e_{AB} gives a higher fraction of quadruples that become dynamically unstable. To get a cleaner view of the effect of eccentricities and to test our code, we performed several calculations where all the explicitly specified parameters of the binaries A (blue solid circle) and B (red open circle) were set to identical values (entries A6e0, A12e0, and A24e0 in Table 1) and we show the results with overlapping circles in Figure 3. Interestingly, the fraction of quadruples composed of two identical binaries that reach $1 - e < 10^{-3}$ starts decreasing considerably only for $a_{AB}/a_A \gtrsim 30$, which is later than for the fiducial case we investigated previously. Due to numerical difficulties with energy

conservation at large relative separations we did not investigate this potentially interesting issue further.

4 COLLISIONS

The KL cycles in triples can change the angular momentum of the inner binary in such a way that it goes from a finite value to essentially zero in a single orbit. As a result, stars that previously were too far from each other to interact through tides or gravitational wave emission will collide (Katz & Dong 2012). However, for equal-mass binaries this occurs only in a few percent of systems where the perturbing body initially orbits almost perpendicularly with respect to the inner binary ($\cos i \sim 0$). In Section 3, we showed that quadruples reach high eccentricity for much larger part of the parameter space than triples. We now investigate whether the same is true for collisions.

In a subset of our calculations, we implemented a collision detection algorithm based on Katz & Dong (2012). Since our calculations are dimensionless, we assume that a clean collision without any previous tidal or gravitational wave interaction occurs when a pericenter distance of the binary is at least by a factor of 4 smaller than any of the previous ones. The eccentricity at this pericenter sets the radius of the stars relative to the semi-major axis. The upper limit on stellar radius for such “clean” collisions to occur is set by the time for quadruple KL cycles to develop, which scales with t_K , relative to the lifetime of the stars or the Hubble time.

Color pixels in Figure 2 show the fraction of binaries that experienced a clean collision for our fiducial calculation. Similar to the cases of eccentricity and orbital flips discussed in Section 3, the clean collision fraction is significant over a large chunk of the parameter space and does not require any fine tuning of the inclinations. In our fiducial case displayed in Figure 2, about 19% and 25% of binaries *A* and *B*, respectively, experience clean collisions. This is much higher than for corresponding triples, where we find 4% and 2%, quite close to the values reported by Katz & Dong (2012) for a similar ratio of semi-major axes. The distribution of $1 - e$ at the moment of clean collisions starts at about 10^{-3} and peaks at about 4×10^{-5} , which permits clean collisions of main-sequence and smaller stars in a Hubble time, as found also by Katz & Dong (2012).

5 DISCUSSION AND CONCLUSIONS

Our numerical calculations of orbital evolution of quadruple systems shows that their KL cycles are qualitatively and quantitatively different from triples. Although the binarity of the KL perturber would seem to be a higher-order effect, it significantly increases the fraction of close passages of stars in quadruple systems compared to equivalent triples. We now explore some of the implications of our findings.

We showed that high eccentricities ($1 - e < 10^{-3}$), orbital flips and stellar collisions occur with a frequency ~ 3 to 12 times higher in quadruples than in triples for $a_{AB}/a_A \sim 10$ to 20 and we estimated that the frequency is still significantly higher at $a_{AB}/a_A \sim 50$. The strength of quadruple KL cycles in both binaries is maximized for equal masses. However, quadruples are also intrinsically less frequent in the field than triples. Thus, to assess the absolute occurrence of high eccentricities in the field population of quadruples and to compare it to triples, we need to know their relative frequencies, and the distributions of semi-major axes and

mass ratios. The best available source is the catalog of Tokovinin (1997, 2008). By analyzing the distance distribution of triples and quadruples composed of two binaries, Tokovinin (2008) found that such quadruples are by a factor of ~ 5 less frequent than triples in the field. The median of a_{AB}/a_A (a_{AB}/a_B) in the Tokovinin (2008) sample is 100 (150) and $\sim 40\%$ ($\sim 30\%$) of quadruples have $a_{AB}/a_A < 50$ ($a_{AB}/a_B < 50$). For triples, the median semi-major axis ratio is ~ 80 and $\sim 45\%$ of systems have a ratio smaller than 50. The medians of q_A , q_B and q_{AB} of 81 quadruple systems in the Tokovinin (2008) sample with mass estimates are 0.64, 0.70, and 0.70, respectively. To summarize, the field sample of quadruples appears to be composed of stars with similar masses and the relative distribution of semimajor axis ratios in quadruples is not significantly different from triples. Although the Tokovinin (2008) catalog is subject to many selection effects, *the lower intrinsic frequency of quadruples in the field is approximately compensated for by the higher efficacy of their KL cycles*. Quadruples are thus an order unity correction to any population synthesis result involving KL cycles in triples (Hamers et al. 2013).

In principle, KL cycles in quadruples will act on both binaries while in a triple only the inner binary will be affected; a quadruple thus potentially produces twice as many “interesting” outcomes. In reality, after the first clean collision, merger, or close encounter in one of the binaries of the quadruple system, this binary will become a single star or its semimajor axis will significantly shrink due to tidal dissipation and the subsequent evolution will then proceed effectively as a triple star, with the usual properties of KL cycles in triples. Assuming that the initial distribution of the ratio of semi-major axis of the inner and mutual orbits was the same in quadruples and triples, the higher efficiency of KL cycles in quadruples evolution will lead to a relative lack of quadruples with small semimajor axes ratio. This is supported by higher median of a_{AB}/a_A and a_{AB}/a_B in the Tokovinin (2008) sample, however a two-way Kolmogorov-Smirnov test on the quadruple and triple semimajor axis ratio distributions does not reveal any statistically significant differences.

The lower frequency of quadruples relative to triples in the field might also be an effect of highly efficient quadruple KL cycles: the fraction of multiple stars in star-forming regions is higher than in the field (e.g. Leinert et al. 1993; Ghez et al. 1997; Kohler & Leinert 1998; Chen et al. 2013) and the frequency of triple and quadruple stars might be comparable (e.g. Correia et al. 2006). Thus, quadruples may be a significant source of triples. Furthermore, stellar evolution and associated mass loss will lead to expansion of orbits that will be more pronounced for the inner binaries than for the mutual orbit and thus yield smaller a_{AB}/a_A and a_{AB}/a_B . On the other hand, when the mass ratio of the binaries evolves farther from unity the quadruple KL cycles become weaker (Section 3.2). Depending on how exactly the mass loss occurs, quadruple systems that did not experience KL cycles due to large separation of the binaries and unfavorable orientation of the orbit for the usual triple KL mechanism might start experiencing strong KL cycles as they evolve (Shappee & Thompson 2013). Additionally, a supernova or a neutron star kick in one member of the quadruple may also lead to interesting results (Shappee & Thompson 2013). Finally, interesting evolution in triple systems of stars and planets occurs also with KL cycles not present (Perets & Kratter 2012; Kratter & Perets 2012).

The evolution of quadruples can produce outcomes that would not be achieved by triples and can involve main sequence stars, white dwarfs, neutron stars, and their combinations. Some outcomes include pairs of close binaries or blue stragglers on a wide

orbit (Perets & Fabrycky 2009), blue stragglers orbiting a close binary, massive white dwarfs orbiting a (close) binary or a blue straggler, etc. Collisions and mergers of stars produce transients that can be observed over great distances. Potential examples include V838 Mon and V1309 Sco (e.g. Soker & Tylenda 2003; Tylenda & Soker 2006; Glebbeek et al. 2008; Glebbeek & Pols 2008; Smith 2011; Tylenda et al. 2011). Additionally, supernovae Ia can occur as a result of a collision of two white dwarfs (e.g. Benz et al. 1989; Rosswog et al. 2009; Raskin et al. 2009, 2010; Lorén-Aguilar et al. 2010; Hawley et al. 2012; Kushnir et al. 2013). The KL mechanism allows such collisions to happen in the field, not only in dense stellar environments, but only a few percent of triples can collide due to severe restrictions on the inclination of the perturbing body (Katz & Dong 2012). However, we find that quadruples produce collisions generically for a large fraction of the parameter space and thus do not require fine tuning in the orientation of the orbits. Furthermore, the quadruple evolution allows for peculiar stars such as blue stragglers, close binaries, or massive white dwarfs to remain at the explosion site. Specifically, our results on KL cycles in quadruples increase by a factor of few the probability of having a metal-poor A star (blue straggler) in the Tycho supernova remnant (Kerzendorf et al. 2012; Thompson & Gould 2012). The higher efficiency of quadruple KL cycles might give important constraints on the future evolution of the known quadruple systems such as those investigated by Tokovinin et al. (2003) and Harmanec et al. (2007). The new dynamics of quadruple systems together with tidal dissipation might explain the 3:2 period ratios of some double eclipsing binaries (Cagaš & Pejcha 2012; Kołaczowski et al. 2013), but more work is required to understand these systems.

Finally, our results imply that stars orbiting a distant stellar binary have significantly higher probability of secular changes to their planetary orbits (Fig. 4) than in systems where the distant body is a point mass (Wu & Murray 2003; Wu et al. 2007; Fabrycky & Tremaine 2007; Naoz et al. 2011, 2013; Katz et al. 2011; Lithwick & Naoz 2011). Without a more detailed investigation of the parameter space, it is not clear whether the enhanced KL cycles in such systems would lead to a higher fraction of hot Jupiters (possibly on retrograde orbits) or more star-planet collisions. Roell et al. (2012) list nine exoplanet host stars that are members of triples that are in the hierarchy considered here. For most of these systems, the separation of the host star from the distant binary relative to its separation is within the range considered in this paper, $a_{AB}/a_B \lesssim 50$. Planets in these systems are not hot Jupiters, but the secular perturbation timescale would be large due to the size of the stellar orbits, which are often visually resolved. A planet subject to quadruple KL cycles on a short timescale would be the progenitor of the hot Jupiter orbiting HD 188753 A, member of a triple stellar system (Konacki 2005). However, this planet was not independently confirmed (Eggenberger et al. 2007).

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REFERENCES

- Antonini, F., & Perets, H. B. 2012, *ApJ*, 757, 27
- Benz, W., Thielemann, F.-K., & Hills, J. G. 1989, *ApJ*, 342, 986
- Blaes, O., Lee, M. H., & Socrates, A. 2002, *ApJ*, 578, 775
- Cagaš, P., & Pejcha, O. 2012, *A&A*, 544, L3
- Chen, X., Arce, H. G., Zhang, Q., et al. 2013, *arXiv:1304.0436*
- Correia, S., Zinnecker, H., Ratzka, T., & Sterzik, M. F. 2006, *A&A*, 459, 909
- Eggenberger, A., Udry, S., Mazeh, T., Segal, Y., & Mayor, M. 2007, *A&A*, 466, 1179
- Fabrycky, D., & Tremaine, S. 2007, *ApJ*, 669, 1298
- Ford, E. B., Kozinsky, B., & Rasio, F. A. 2000, *ApJ*, 535, 385
- Fregeau, J. M., Cheung, P., Portegies Zwart, S. F., & Rasio, F. A. 2004, *MNRAS*, 352, 1
- Galassi, M. et al, *GNU Scientific Library Reference Manual* (3rd Ed.), ISBN 0954612078
- Ghez, A. M., McCarthy, D. W., Patience, J. L., & Beck, T. L. 1997, *ApJ*, 481, 378
- Glebbeek, E., Pols, O. R., & Hurley, J. R. 2008, *A&A*, 488, 1007
- Glebbeek, E., & Pols, O. R. 2008, *A&A*, 488, 1017
- Hamers, A. S., Pols, O. R., Claeys, J. S. W., & Nelemans, G. 2013, *MNRAS*, 430, 2262
- Harmanec, P., Mayer, P., Prša, A., et al. 2007, *A&A*, 463, 1061
- Hawley, W. P., Athanassiadou, T., & Timmes, F. X. 2012, *ApJ*, 759, 39
- Heggie, D. C. 1974, *Celestial Mechanics*, 10, 217
- Holman, M., Touma, J., & Tremaine, S. 1997, *Nature*, 386, 254
- Innanen, K. A., Zheng, J. Q., Mikkola, S., & Valtonen, M. J. 1997, *AJ*, 113, 1915
- Katz, B., Dong, S., & Malhotra, R. 2011, *Physical Review Letters*, 107, 181101
- Katz, B., & Dong, S. 2012, *arXiv:1211.4584*
- Kerzendorf, W. E., Yong, D., Schmidt, B. P., et al. 2012, *arXiv:1210.2713*
- Kohler, R., & Leinert, C. 1998, *A&A*, 331, 977
- Kołaczowski, Z., Mennickent, R. E., Rivinius, T., & Pietrzyński, G. 2013, *MNRAS*, 429, 2852
- Konacki, M. 2005, *Nature*, 436, 230
- Kozai, Y. 1962, *AJ*, 67, 591
- Kratter, K. M., & Perets, H. B. 2012, *ApJ*, 753, 91
- Kushnir, D., Katz, B., Dong, S., Livne, E., & Fernández, R. 2013, *arXiv:1303.1180*
- Leinert, C., Zinnecker, H., Weitzel, N., et al. 1993, *A&A*, 278, 129
- Lidov, M. L. 1962, *Planet. Space Sci.*, 9, 719
- Lithwick, Y., & Naoz, S. 2011, *ApJ*, 742, 94
- Lorén-Aguilar, P., Isern, J., & García-Berro, E. 2010, *MNRAS*, 406, 2749
- Mazeh, T., & Shaham, J. 1979, *A&A*, 77, 145
- Mikkola, S. 1985, *MNRAS*, 215, 171
- Miller, M. C., & Hamilton, D. P. 2002, *ApJ*, 576, 894
- Naoz, S., Farr, W. M., Lithwick, Y., Rasio, F. A., & Teyssandier, J. 2011, *Nature*, 473, 187
- Naoz, S., Kocsis, B., Loeb, A., & Yunes, N. 2012, *arXiv:1206.4316*
- Naoz, S., Farr, W. M., & Rasio, F. A. 2012, *ApJ*, 754, L36

- Naoz, S., Farr, W. M., Lithwick, Y., Rasio, F. A., & Teyssandier, J. 2013, MNRAS, accepted, arXiv:1107.2414
- Nesvorný, D., Alvarellos, J. L. A., Dones, L., & Levison, H. F. 2003, AJ, 126, 398
- Perets, H. B., & Fabrycky, D. C. 2009, ApJ, 697, 1048
- Perets, H. B., & Kratter, K. M. 2012, ApJ, 760, 99
- Raskin, C., Timmes, F. X., Scannapieco, E., Diehl, S., & Fryer, C. 2009, MNRAS, 399, L156
- Raskin, C., Scannapieco, E., Rockefeller, G., et al. 2010, ApJ, 724, 111
- Roell, T., Neuhäuser, R., Seifahrt, A., & Mugrauer, M. 2012, A&A, 542, A92
- Rosswog, S., Kasen, D., Guillochon, J., & Ramirez-Ruiz, E. 2009, ApJ, 705, L128
- Shappee, B. J., & Thompson, T. A. 2013, ApJ, 766, 64
- Smith, N. 2011, MNRAS, 415, 2020
- Soker, N., & Tylenda, R. 2003, ApJ, 582, L105
- Thompson, T. A. 2011, ApJ, 741, 82
- Thompson, T. A., & Gould, A. 2012, arXiv:1210.6050
- Tokovinin, A. A. 1997, A&AS, 124, 75
- Tokovinin, A., Balega, Y. Y., Pluzhnik, E. A., et al. 2003, A&A, 409, 245
- Tokovinin, A., Thomas, S., Sterzik, M., & Udry, S. 2006, A&A, 450, 681
- Tokovinin, A. 2008, MNRAS, 389, 925
- Tylenda, R., & Soker, N. 2006, A&A, 451, 223
- Tylenda, R., Hajduk, M., Kamiński, T., et al. 2011, A&A, 528, A114
- Wu, Y., & Murray, N. 2003, ApJ, 589, 605
- Wu, Y., Murray, N. W., & Ramsahai, J. M. 2007, ApJ, 670, 820